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Sensitivity-Based Chance-Constrained Generation Expansion Planning

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Abstract

A Generation Expansion Planning problem with load uncertainty is formulated based on joint chance-constrained programming (CCP) and is solved by incorporating sensitivity into iterative algorithms. These algorithms exploit the characteristics of the system and its response to load variations. Sensitivities help to classify buses according to stress level, and sensitivity-based iterative algorithms distinguish each bus based on its contribution to the overall system reliability. The use of sensitivity overcomes some of the mathematical obstacles to solving joint CCP problems and, in addition, leads to optimal expansion solutions because uncertain loads are correctly estimated. The IEEE 30- and 118-bus test systems are used to demonstrate the proposed algorithms, and the results of these algorithms are compared with those of other algorithms for solving the joint CCP problem.

Keywords: Generation expansion planning, chance-constrained programming, sensitivity.

1. Introduction

Today's power systems, be they regulated or deregulated, are exposed to ever more sources of uncertainty, such as the integration of renewable sources, demand participation, and generation and transmission availability. This uncertainty and the increasing demand for power raise new challenges for utility planners, whose goal is to provide reliable power to consumers at the lowest possible cost.

Generation Expansion Planning (GEP) models are used to determine the size, type, and location of the additional units required to satisfy the forecasted demand. GEP models are often sensitive to uncertainty, so neglecting uncertainty may lead to unrealistic solutions. Thus, stochastic-optimization approaches are used to incorporate uncertainty into GEP models. A deterministic multiperiod and multi-objective GEP is solved in [18]. In [20], a GEP model with uncertain demand is formulated and solved by using stochastic dynamic programming. Different applications have led to different types of stochastic-optimization models. One such model is the recourse-based model [8] in which, in a first step, optimal decisions are taken and, after some of the uncertainty is resolved, a recourse is available to re-optimize. Another model is the expected-value model, which minimizes the expected value of the cost subject to the expected values of constraints [8, 14]. Yet another way to handle uncertainty in probabilistic terms is chance-constrained programming (CCP).

The contribution of this paper is to incorporate sensitivity into the iterative algorithms used to solve the GEP problem with load uncertainty for a vertically integrated power system modeled by using joint CCP. The proposed iterative algorithm distinguishes between stressed and nonstressed buses in the system, and the iterative updates for each bus are different. Although joint CCP has several advantages, it is not widely used because of the mathematical challenges involved. This work addresses several of those challenges and differs from other similar algorithms [27, 19] in the way that different chance constraints are treated differently as appropriate. The present study builds on [21], which only separated stressed buses: herein we consider information from both stressed and nonstressed buses.

The rest of the paper is structured as follows: Section 2 briefly describes expansion planning models and the optimization approaches that they use, with an emphasis on the CCP approach. Section 3 uses CCP to formulate the GEP problem with load uncertainty. Section 4 discusses previous iterative procedures to solve the CCP formulation, and Section

5 explains the improved algorithms proposed herein that incorporate sensitivities. Section 6 reports and analyzes the computational results of applying the algorithms to two standard test systems. Section 7 concludes the paper.

2. Background

Power system planning has become an intensive process and the investment in utility planning is significant [22]. A survey of the optimization techniques used in utility planning is given in [12], and of the several stochastic optimization techniques used to model uncertainty, only robust optimization [6, 7] and CCP explicitly aim to achieve a prescribed reliability level. When the probability distribution of the uncertain random variable is known, CCP is a particularly suitable optimization technique for including uncertainty in the solution. Although the GEP problem contains several sources of uncertainty, we concentrate in this work on demand uncertainty, which is the main source of uncertainty. For power-system demand, the probability distribution can be obtained by using historic data.

Unlike in deterministic optimization where all the constraints have to be satisfied, CCP allows some or all of the constraints to be satisfied only with a given probability. It was first introduced in [1] and has been applied extensively to a wide range of engineering, financial, and management applications. Originally, CCP was used as an analytical tool for planning problems because it explicitly incorporates risk.

Since then, CCP has been applied to power-system planning and operation problems. A generation-planning model was introduced in [11], where a probabilistic reliability criterion was considered for both discrete and continuous random generation. In [28], a CCP-based formulation of transmission expansion planning was solved by using a genetic algorithm. The effect of wind uncertainty in transmission expansion planning was discussed in [30], in which the authors modeled wind uncertainty with a probability density function and the resulting CCP-based problem was solved by using a genetic algorithm. In [13], a generation and transmission expansion problem was modeled by using two-stage stochastic programming, in which a risk factor is introduced into the objective function. The solution algorithm was based on the minimum-variance approach [16], which minimizes the risk in an investment project. A market-based generation and transmission expansion planning model was solved in [25] by using scenario-based formulation and Monte Carlo simulation (MCS). In that work, a reduction technique was applied to reduce the number of scenarios considered. A GEP problem for vertically integrated systems with load uncertainty was modeled by using CCP and solved with a modified iterative algorithm in [19]; this approach proved to have fewer iterations. CCP has also been applied to operation and stability problems; for example, a unit-commitment problem was modeled by using CCP in [3] and was solved iteratively.

These previous applications of CCP to power system problems present two drawbacks: separate chance constraints are used [11, 29] where only one probabilistic constraint is relevant, and the solution approach for joint CCP is computationally costly. Problems based on joint CCP are difficult to solve and therefore are usually transformed into equivalent deterministic approximations, as proposed in [2]. In the present work, these approximations and thus the resulting iterative algorithms are improved by including sensitivities, which leads to better solutions.

3. Generation Expansion Planning under Uncertainty

The main objective in GEP is to minimize cost subject to the operational constraints of the system. Mathematically, this can be expressed by using a modified version of the following formulation [11]:

$$\min \sum_{i=1}^{n_{\text{bus}}} w_i C_{b,i}^{\text{n}} p_{g,i}^{\text{n,max}} + \sum_{i=1}^{n_{\text{bus}}} C_{p,i}^{\text{n}} p_{g,i}^{\text{n}} + \sum_{i=1}^{n_{\text{bus}}} C_{p,i}^{\text{e}} p_{g,i}^{\text{e}}, \tag{1a}$$

s.t.

$$p_{g,i}^{\text{n}} + p_{g,i}^{\text{e}} - p_{s,i} = p_i^{\text{L}},\ i = 1, \ldots, n_{\text{bus}}, \tag{1b}$$

$$p_{s,i} = \sum_j -b_{i,j}(\delta_i - \delta_j), i = 1, \ldots, n_{\text{bus}}, \tag{1c}$$

$$p_{s,i}^{\min} \leq p_{s,i} \leq p_{s,i}^{\max},\ i = 1, \ldots, n_{\text{bus}}, \tag{1d}$$

$$p_{g,i}^{\text{e,min}} \leq p_{g,i}^{\text{e}} \leq p_{g,i}^{\text{e,max}},\ i = 1, \ldots, n_{\text{bus}}, \tag{1e}$$

$$w_i p_{g,i}^{\text{n,min}} \leq p_{g,i}^{\text{n}} \leq w_i p_{g,i}^{\text{n,max}},\ i = 1, \ldots, n_{\text{bus}}, \tag{1f}$$

$$w_i \in \{0, 1\}, \tag{1g}$$

where

$C^{\mathrm{n}}_{b,i}, C^{\mathrm{n}}_{p,i}$ are the investment and production cost of a new unit at bus i;

$C^{\mathrm{e}}_{p,i}$ is the production cost of an existing unit at bus i;

$p^{\mathrm{n}}_{g,i}$, $p^{\mathrm{e}}_{g,i}$ are the active power levels of respectively new and existing units g at bus i;

$p^{\mathrm{n,min}}_{g,i}$ and $p^{\mathrm{n,max}}_{g,i}$ are the minimum and maximum active power levels of a new unit g at bus i;

$p^{\mathrm{e,min}}_{g,i}$ and $p^{\mathrm{e,max}}_{g,i}$ are the minimum and maximum active power levels of existing units g at bus i;

p^{L}_{i} is the load connected to bus i;

$p_{s,i}$ is the net power flow in all the lines connected to bus i;

$p^{\mathrm{min}}_{s,i}$ and $p^{\mathrm{max}}_{s,i}$ are the minimum and maximum net power flow in lines connected to bus i;

δ_i is the voltage phase angle at bus i

$b_{i,j}$ is the susceptance of the line between buses i and j;

n_{bus} is the number of buses in the system;

w_i is the binary decision variable for new generation at bus i.

The objective function (1a) is the total of investment and operation costs, the constraint (1b) is the real-power balance equation, and Eq. (1c) is the sum of the line flows over all lines connected to bus i. Constraints (1d)–(1f) give the operational limits for the transmission lines and generating units. Finally, constraint (1g) expresses the binary nature of the decision variable w_i.

The constraints (1b) and (1c) are the DC power flow equations. Although there are models for transmission network expansion planning that enable the use of AC power flow equations [31, 24] and could arguably be used here, we chose to use the DC formulation because our algorithms are for the initial stages of planning that are carried out several years before the actual situation, and the DC approximation suffices for this purpose. If desired, an AC power flow can be computed afterwards to confirm feasibility.

Note that the above formulation ignores load uncertainty. The load represented here is the average forecasted load. The loading level appears in Eq. (1b), so to include uncertainty in probabilistic terms, we change constraint (1b) to

$$\text{Prob}\left(\bigcap_{i=1}^{n_{\text{bus}}}(p_{g,i}^{\text{n}} + p_{g,i}^{\text{e}} - p_{s,i} \geq p_i^{\text{L}})\right) \geq \alpha, \tag{2}$$

where α is a user-defined probability threshold that represents the confidence level (or reliability level). Traditional GEP usually uses the loss of load expectation or the installed reserve margin as the reliability criterion. However, it was shown in [11] that modeling transmission-line constraints in terms of reliability criteria is advantageous in GEP. If the optimization becomes infeasible with this type of modeling and load curtailment is not allowed, a transmission upgrade is needed in the first stage of planning. This helps to identify the equilibrium conditions whereby the demand is matched with installed generation and does not account for any contingency studies. The left-hand side of Eq. (2) is a joint probability, i.e., it is the probability of n_{bus} events occurring simultaneously; thus constraint (2) is a joint chance constraint [23]. This probability can be calculated by numerical integration, but this technique is limited by dimensionality [23] and is only possible in practice for small problems.

A practical means to address this difficulty is to convert the joint chance constraint (2) into a set of individual constraints. First, observe that Eq. (2) can be reformulated as

$$\text{Prob}\left\{\bigcup_{i=1}^{n_{\text{bus}}}(p_{g,i}^{\text{n}} + p_{g,i}^{\text{e}} - p_{s,i} \geq p_i^{\text{L}})^c\right\} \leq 1 - \alpha \tag{3}$$

where the exponent c denotes the complementary event (in this case, the inequality inside the parentheses not holding), because the sum of the probabilities of an event and of its complementary event is 1. Now we can write a set of sufficient conditions for Eq. (3) to hold:

$$\text{Prob}\left\{(p_{g,i}^{\text{n}} + p_{g,i}^{\text{e}} - p_{s,i} \geq p_i^{\text{L}})^c\right\} \leq \frac{1-\alpha}{n_{\text{bus}}}, \quad i = 1, \ldots, n_{\text{bus}}. \tag{4}$$

Therefore, the probabilistic real-power flow equation (2) can be approximated as

$$\text{Prob}\left\{p^{\text{n}}_{g,i} + p^{\text{e}}_{g,i} - p_{s,i} \geq p^{\text{L}}_i\right\} \geq 1 - \frac{1-\alpha}{n_{\text{bus}}}, \qquad i = 1, \ldots, n_{\text{bus}}, \tag{5}$$

where (5) holds because the two probabilities in (4) and (5) add up to 1. In short, the constraint (2) can be replaced with the stronger set of constraints (5).

If the probability distribution of the load is known, each of the constraints in Eq. (5) can be replaced by its deterministic equivalent. For GEP, it is reasonable to assume that the load at bus i follows a normal distribution [15]. If μ^{L}_i denotes its mean and σ^{L}_i denotes its variance, then we can rewrite the constraints (5) as

$$\text{Prob}\left\{\frac{p^{\text{n}}_{g,i} + p^{\text{e}}_{g,i} - p_{s,i} - \mu^{\text{L}}_i}{\sigma^{\text{L}}_i} \geq \frac{p^{\text{L}}_i - \mu^{\text{L}}_i}{\sigma^{\text{L}}_i}\right\} \geq 1 - \frac{1-\alpha}{n_{\text{bus}}}, \quad i = 1, \ldots, n_{\text{bus}}. \tag{6}$$

Because $(p^{\text{L}}_i - \mu^{\text{L}}_i)/\sigma^{\text{L}}_i$ is a standard normal random variable, Eq. (6) is equivalent to

$$\phi\left\{\frac{p^{\text{n}}_{g,i} + p^{\text{e}}_{g,i} - p_{s,i} - \mu^{\text{L}}_i}{\sigma^{\text{L}}_i}\right\} \geq 1 - \frac{1-\alpha}{n_{\text{bus}}}, \tag{7}$$

where ϕ is the normal probability distribution function. Finally, let Z_α be the inverse cumulative distribution of $1 - (1-\alpha)/n_{\text{bus}}$, i.e., $Z_\alpha = \phi^{-1}\{1 - (1-\alpha)/n_{\text{bus}}\}$. Then, Eq. (7) can be rewritten as

$$\frac{p^{\text{n}}_{g,i} + p^{\text{e}}_{g,i} - p_{s,i} - \mu^{\text{L}}_i}{\sigma^{\text{L}}_i} = Z_\alpha, \tag{8}$$

or equivalently,

$$p^{\text{n}}_{g,i} + p^{\text{e}}_{g,i} - p_{s,i} = \mu^{\text{L}}_i + \sigma^{\text{L}}_i Z_\alpha. \tag{9}$$

The remainder of this paper is concerned with improving the choice of Z at each bus so as to reduce the the total cost of implementing the resulting generation expansion plan. In particular, although the derivation detailed above assumes that the load uncertainty is the same for every bus, this is generally not the case. Therefore, it makes sense to adjust, or update, Z for each bus; doing so reduces the cost of achieving the required level of reliability.

4. Previously Proposed Z-Update Algorithms

4.1. Uniform Z-update algorithm

Given the desired probability level α, the general framework for solving the GEP problem by using the deterministic formulation described above is as follows:

(1) A set of new generations is computed by solving the mixed-integer linear programming (MILP) problem (1a)–(1g) with the constraint (1b) replaced by the approximate deterministic equivalent (9).

(2) Sample load scenarios are randomly generated by a MCS, and the optimal power flow (OPF) is obtained based on the solution from step (1). Because the new generation is fixed, w_i is not a variable and there is no investment cost in the objective function. The number of feasible cases divided by the total number of scenarios gives the estimate of the probability.

(3) If the estimated probability is not acceptable, then the Z value is updated and the process is repeated from step (1) until the target probability is achieved.

The update of Z implies a change in the load conditions p_i^{L} and is likely to significantly impact the new optimal generation decisions. The Z-update method proposed in [27] is based on interpolating the univariate and multivariate variables and the false-position method [17]. The steps in this algorithm are as follows:

(1) The GEP problem is initially solved twice with values Z^{Hi} and Z^{Lo} that are chosen so that Z^{Lo} is a value below Z^{α} obtained by generating a small random deviation from Z^{α}, and Z^{Hi} is a random value above Z^{α} also chosen using a small random deviation. Thus Z^{Hi} and Z^{Lo} correspond to probabilities p^{Hi} and p^{Lo}, which are, respectively, higher and lower than the desired probability α.

(2) The probabilities p^{Hi} and p^{Lo} are determined by obtaining OPFs for various normally distributed load samples generated by a MCS. These probabilities are converted to the corresponding univariate-space Z equivalents Z_1 and Z_2 by using the probability distribution function of the random variable.

(3) Based on these univariate Z and multivariate probability values, Z_α is updated for the next iteration by using the formula

$$Z_\alpha \leftarrow Z^{\mathrm{Lo}} + \left[\frac{Z_\alpha - Z_2}{Z_1 - Z_2}\left(Z^{\mathrm{Hi}} - Z^{\mathrm{Lo}}\right)\right]. \tag{10}$$

The iterative algorithm basically tries to shrink the interval $[Z^{\mathrm{Hi}}, Z^{\mathrm{Lo}}]$. We refer to it as the uniform Z-update algorithm because the same Z value is used to update the constraint (9) for every bus.

(4) The GEP problem is solved with the updated Z_α, then the OPF is obtained to determine the probability P_{feas} of the new solution and the corresponding Z^{n} is calculated.

(5) If $\mid P_{\mathrm{feas}} - \alpha \mid \leq \triangle\alpha$, then the algorithm terminates. Here, $\triangle\alpha$ is a small tolerance allowed in the target probability. Whenever $P_{\mathrm{feas}} > \alpha \pm \triangle\alpha$, i.e., the tolerance is not satisfied, the false-position method [17] is used to choose new lower and higher values for Z:

   (a) If $Z^{\mathrm{n}} < Z_\alpha$ then Z^{Lo} and Z_2 are replaced with the new Z_α and Z^{n}, respectively.

   (b) If $Z^{\mathrm{n}} > Z_\alpha$ then Z^{Hi} and Z_1 are replaced with the new Z_α and Z^{n}, respectively.

(6) The process is repeated until the target probability is reached.

Thus, by updating the Z value, the loading conditions and the expansion decisions are modified to reach the target confidence level.

4.2. Sensitivity-based Z update and stressed-bus-classification–based Z update algorithm

In the uniform Z-update algorithm, Z is updated uniformly for every bus. Thus, the individual impact of each bus on the system reliability is not considered. This drawback of this algorithm was addressed in our earlier paper [21] by using bus sensitivity. By focusing on the impact of the stressed buses, the cost was reduced.

The outline of this algorithm is similar to that of the uniform Z-update algorithm. The main difference is the identification of the stressed buses and updating Z independently for each bus. Given the desired probability α, the steps of the algorithm are the same except for a modification in Eq. (10). The modified steps for this algorithm are as follows:

(1) After applying a MCS, the success probability achieved with the current expansion solution is known. The infeasible OPF cases imply that at least one bus has insufficient generation to meet the load. These are the stressed buses of the system. To identify them, we solve a different GEP problem with a positive "slack" variable and a corresponding penalty is introduced for each bus:

$$\min \sum_{i=1}^{n_{\text{bus}}} C_{p,i}^{\text{n}} p_{g,i}^{\text{n}} + \sum_{i=1}^{n_{\text{bus}}} C_{p,i}^{\text{e}} p_{g,i}^{\text{e}} + \sum_{i=1}^{n_{\text{bus}}} C^{\text{pen}} s_i, \tag{11a}$$

s.t.

$$p_{g,i}^{\text{n}} + p_{g,i}^{\text{e}} - p_{s,i} = p_i^{\text{L}} - s_i,\ i = 1, \dots, n_{\text{bus}}, \tag{11b}$$

$$s_i \geq 0, \quad i = 1, \dots, n_{\text{bus}}, \tag{11c}$$

$$\text{Constraints(1c)–(1f)}, \tag{11d}$$

where C^{pen} is the penalty cost for the slack variables. Provided that C^{pen} is chosen sufficiently large, the slack variables will be nonzero only when no other way exists to make the OPF feasible, and only at those buses where reducing the load makes the OPF feasible.

(2) The Z_α^i value for bus i is then updated as

$$Z_\alpha^i \leftarrow Z^{\text{Lo}} + \left[\frac{Z_\alpha - Z_2}{Z_1 - Z_2} \left(Z^{\text{Hi}} - Z^{\text{Lo}} \right) \right] \left(\frac{\bar{s}_i}{\hat{s}} \right), \tag{12}$$

where $\bar{s}_i$ is the sum of the slack variables for bus i over all MCS scenarios, and $\hat{s} = \max_{i=1,\dots,n_{\text{bus}}} \bar{s}_i$.

The idea is that the ratio $\bar{s}_i/\hat{s}$ reflects the relative failure rate of bus i. In particular, Z is increased for stressed buses according to the magnitude of the ratio, and it is maintained at Z^{Lo} for nonstressed buses.

(3) The GEP problem is again solved by using the new Z values, and the process is repeated until the target probability is reached.

The results in [21] show that this approach lowers the GEP total cost with respect to the uniform Z-update method.

4.3. Conceptual example contrasting the two previously proposed Z-update algorithms

To illustrate the difference between the algorithms in Sections 4.1 and 4.2, consider a system such as the Nordic32 test system [10]. This test system, developed by CIGRE task force 38-02-08, was inspired by the Nordic Power System, and can be generally characterized as a generation-rich area in the north, while the demand is greater than generation in the central and southern regions. From a stability point of view, transmission limits generally restrict the power flow from north to south. For the sake of simplicity, the system can be considered a two-bus system (north and south/central). Using this system as a base for a long-term planning problem, the uniform Z-update algorithm (where demand is increased throughout the system) will lead to increased generation in both the north and the south, even though there is no need for generation in the north and there are significant transmission constraints to transmit power out of that region. Incorporating slack variables to determine stressed and non-stressed buses provides a means to indicate that relative to the north, increased generation is needed in the other regions. By allowing an increased emphasis on the demand in the central and southern regions, generation will be installed in these regions and unnecessary generation is not added to an already generation-rich north.

5. Improved Z-Update Algorithms

We now study further ways to improve the choice of Z, and thereby create improved algorithms for solving the chance-constrained GEP problem.

5.1. Nonstressed-Bus-Classification–Based Z Update

The algorithm in Section 4.2 aims to increase Z in the chance constraints corresponding to stressed buses. Similarly, certain buses in the system could have the lowest impact on the overall system reliability. For these buses, the loading level can be reduced, so Z^{Lo} can be reduced. In this way, the total cost can be further reduced for the same confidence level.

To determine the buses that are least sensitive to load fluctuations, slack variables are again used but in a different way. Although the slack variables in Section 4.2 reduce the load at a bus, these new slack variables attempt to increase the load at each bus. We therefore refer to them as "negative-slack" variables. These negative-slack variables are added to the real-power-balance constraint of the feasible OPF scenarios. They serve to increase the load for certain buses such that the OPF remains feasible after the increase. In this way, the additional load each bus can accept is quantified without affecting the reliability of the system. In other words, the higher the value of the negative-slack variable for a particular bus, the more the bus is nonstressed.

The steps in this algorithm are as follows:

(1) The GEP problem is solved for a Z^{Lo} chosen so that the probability of success with Z^{Lo} is much lower than the target probability.

(2) After determining the probability of success by using normally distributed load samples, the feasible samples are selected and their loads are modified by using the following optimization problem where the negative-slack variables have been introduced:

$$\max \sum_{i=1}^{n_{\mathrm{bus}}} m_i, \tag{13a}$$

s.t.

$$p_{g,i}^{\mathrm{n}} + p_{g,i}^{\mathrm{e}} - p_{s,i} = p_i^{\mathrm{L}} + m_i,\ i = 1, \ldots, n_{\mathrm{bus}}, \tag{13b}$$

$$m_i \geq 0, \quad i = 1, \ldots, n_{\mathrm{bus}}, \tag{13c}$$

$$\text{Constraints (1c)} - \text{(1f)}. \tag{13d}$$

Note that the objective is to maximize the sum of the negative-slack variables, i.e., to determine the maximum-load point of each bus. A nonzero optimal m_i indicates that bus i has the capacity to handle a higher load. Thus, the negative-slack values reflect the sensitivity of the buses to the loading conditions.

(3) The nonstressed buses identified in the previous step are the candidate buses for decreasing Z. The Z value for each bus i is thus updated as follows:

$$Z_\alpha^i \leftarrow Z^{\mathrm{Lo}} \left(1 - \frac{\bar{m}_i}{\hat{m}}\right), \tag{14}$$

where $\bar{m}_i$ is the sum over all the scenarios of the negative-slack variables for bus i, and

$$\hat{m} = \max_{i=1,\ldots,n_{\mathrm{bus}}} \bar{m}_i.$$

Similar to the ratio $\bar{s}_i/\hat{s}$ in Eq. (12), the ratio $\bar{m}_i/\hat{m}$ reflects the relative stress level of bus i. The buses that can accept the largest additional load have a ratio of unity, whereas the buses unable to handle any additional load have a ratio of zero.

(4) The GEP problem is again solved with the updated Z, and the process is repeated until the target probability is achieved.

5.2. Combined-bus-classification–based Z-update approach

Because both the stressed and nonstressed bus-classification–based Z approaches are advantageous compared with the uniform Z-update method, we now consider combining both approaches into a single approach.

First, after running the initial MCS to compute the success probability, the infeasible and feasible scenarios are separated. The load samples are modified by using the respective slack variables and an optimization is run. Based on these results, both the stressed and nonstressed buses are determined. This classification is done after the first iteration. Once the classification is made, it is maintained for all other iterations to ensure that Z does not oscillate around Z^{Lo}. A bus that was neither stressed nor nonstressed in the first iteration can become either stressed or nonstressed in subsequent iterations. However, we ensure that if a bus was a stressed in the first iteration, it cannot later become a nonstressed bus (and vice versa) by not allowing the slack variables at that bus to change in subsequent iterations.

The Z value for the stressed (nonstressed) buses is increased (decreased). Thus, this combined-bus-classification method updates Z in both directions.

The steps in the combined-bus-classification–based Z-update approach are thus as follows:

(1) The GEP problem is solved for two initial values Z^{Lo} and Z^{Hi}.

(2) Given the initial points Z^{Lo} and Z^{Hi}, the probability of success is determined by using a MCS and obtaining the OPF. The probabilities must be less (more) than the target probabilities p^{Lo} (p^{Hi}).

(3) The infeasible and feasible samples are grouped separately. The infeasible samples are optimized as described by Eqs. (11a)–(11d), and the feasible samples are optimized

according to Eqs. (13a)–(13d). Thus the type of bus and its stress level are determined by the slack value and the slack ratio.

(4) From the previous step, the bus classification is known and maintained throughout the process. New buses can be added to this classification during the iteration process, but modifications of the existing classification are not allowed. Based on this classification,

    if ($s_i \neq 0$)

    Z is updated by using Eq. (12)

    elseif ($m_i \neq 0$)

    Z is updated by using Eq. (14).

(5) With the new Z for each bus, the GEP problem is solved again. A line search is used in to update the stressed buses, so bounds are swapped after every update. The iterative process is repeated until the target probability is reached.

6. Computational results

The proposed algorithms were implemented and applied to the IEEE 30-bus and IEEE 118-bus test systems [4] and the computational results are given herein. The base loading levels were the same as given in the test system. A reserve capacity of 5% was added to the load for both systems. For the IEEE 30-bus system, a load growth of 3% per year was assumed, whereas the load growth was assumed to be a one-time increase of 40% for the 118-bus system. The latter choice demonstrates that the algorithms can work even for a multiyear formulation. The uncertainty in the load was modeled as a normal distribution satisfying the three standard-deviation criteria $3\mu = 0.25\sigma$, where μ and σ are the mean and standard deviation of the normalized load growth.

The line limits of the 30-bus system were modified as follows: The transfer capacity of the line connecting buses 5 and 7 was reduced to 0.45 from 0.7, and the capacity of the line between buses 6 and 8 was reduced to 0.28 from 0.32. The results for the 30-bus system were computed with these modified line limits. The 118-bus system had sufficient transmission-line capacity, so no modifications were made to this system.

The simulation uses the system configuration, transmission-line characteristics, generator-capacity limits, and transmission-line flow limits given for the test system. The installation cost for a new unit is 260 000 \$/MW [5], and the production cost for both existing and new generation units is 45 \$/MWh [5]. Another scenario that considers different generation technologies and thus different investment and operation costs is also shown. The MCS generates 1000 samples of normally distributed demand per run. The iterative algorithm stops when P_{feas} is within $\pm 0.5\%$ of the target probability. The MILP problems were solved by using the BONMIN [9] solver accessed via GAMS [26]. The computations were done on a personal computer, and the computation time was less than 1 hour for all the approaches.

Figures 1 and 2 show the frequency distribution of the load samples at a node of the system. These samples are used to compare the generation expansion solution at the end of the iterative procedure with different algorithms for the IEEE 30- and 118-bus test systems, respectively. Tables 1 and 2 show the variation in the number of new units built according to the proposed algorithms and also compares the results with the stressed-bus-classification method.

Fewer new units are built with the proposed algorithms, so the investment cost decreases, as shown in Figs. 3 and 4. The variation in investment cost reveals that the cost increases with increasing reliability level. Thus, CCP helps to inform users of the extra cost involved to increase the reliability of the system. Also, the results indicate that the combined bus-classification method involves the least investment cost. When compared with the uniform method, stressed-bus classification will always result in reduced cost because sensitivity is included. The other two methods try to reduce the expansion cost by incorporating different forms of sensitivity. Sometimes, an additional generation unit may be necessary, so no other expansion solution is better and the same investment cost is obtained.

This reduction in investment cost is possible because of the variation in updating Z. The Z-update variations for the IEEE 30- and 118-bus test systems are shown in Figs. 5 and 6, respectively. The results indicate that, for stressed-bus classification, Z increases for the identified stressed buses. Similarly, for the nonstressed-bus-classification method,

Z decreases for the identified nonstressed buses. The combined-bus-classification method updates Z in both directions, i.e., it increases Z for stressed buses and decreases Z for nonstressed buses.

To further show the effectiveness of the above algorithms, we tested them on an IEEE 30-bus test system under different scenarios. A power system will contain different generation technologies, each with different installation and operation costs. Here, two different generation technologies (coal and gas) are considered. The installation cost of a coal (gas) plant is taken to be 260 000 (120 000) \$/MW [5]. The operation cost of a coal (gas) plant is taken to be 45 (84) \$/MWh [5]. In general, gas plants have lower installation costs but higher operation costs so they are used for peak loads, while coal plants are used as base plants. For the candidate buses considered for new generation units, any type of unit can be installed. Among the six existing generation units, three units were considered to be coal type and the remaining three were considered to be gas type. The generators for buses 1, 2, and 13 were considered to be coal type and the generators for buses 22, 23, and 27 were considered to be gas type. The other parameters used for the MCS were the same as in Section 6, and the same load samples were used (see Fig. 1).

Table 3 gives the variation in the number of new units installed for different confidence levels and proposed Z-update algorithms, and Fig. 7 shows the variation in the corresponding investment costs. As expected, fewer new units are built by using the stressed-bus-classification–based Z-update method than by using the uniform Z-update method because of the identification of stressed buses and the incorporation of the corresponding sensitivity. Although the installation costs of gas units are lower, the operation costs are higher, so no gas units are installed until a 94% confidence level is reached in the stressed-bus-classification–based Z-update method. Using the nonstressed-bus-classification–based and combined-bus-classification–based Z-update methods results in no change in the installation of new generations. This is again due to the higher operation costs of gas units. Although the expansion costs do not decrease, neither do they increase, which proves again that these methods are more beneficial than the uniform Z-update method.

7. Conclusion

The GEP problem with load uncertainty was modeled by using chance-constrained programming. This model was solved by using new iterative-solution algorithms that identify a system's stressed and nonstressed buses. The algorithms emphasize or de-emphasize certain buses in the system depending on the response of the system to load. Computational results for the IEEE 30- and 118-bus systems are presented and compared with the results of previous approaches. The results indicate that generation expansion solutions obtained by using the proposed approach deliver the required reliability at lower cost. This work can be extended to any joint application.

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Figure Captions:

Fig. 1. Frequency distribution of load samples for the IEEE 30-bus system.
Fig. 2. Frequency distribution of load samples for the IEEE 118-bus system.
Fig. 3. Variation in investment cost for IEEE 30-bus system.
Fig. 4. Variation in investment cost for IEEE 118-bus system.
Fig. 5. Z compared for IEEE 30-bus system and 92% target probability.
Fig. 6. Z compared for IEEE 118-bus system and 92% target probability.
Fig. 7. Comparison of investment cost for GEP considering different generation technologies for IEEE 30-bus test systems.

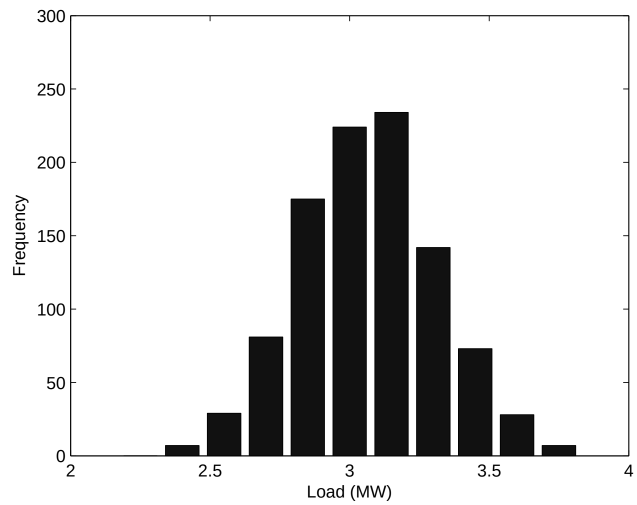

Figure 1: Frequency distribution of load samples for the IEEE 30-bus system.

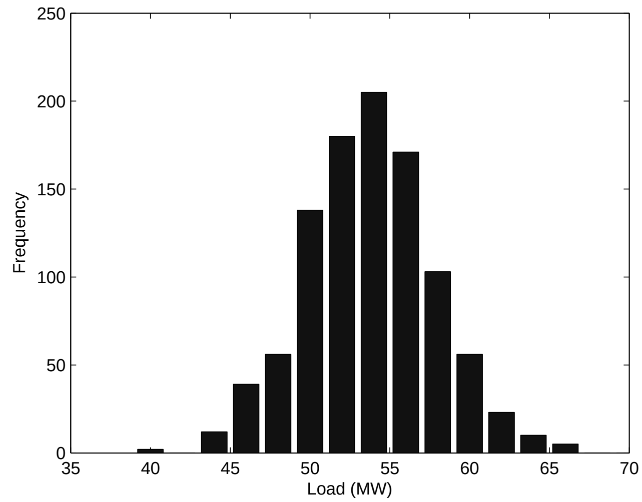

Figure 2: Frequency distribution of load samples for the IEEE 118-bus system.

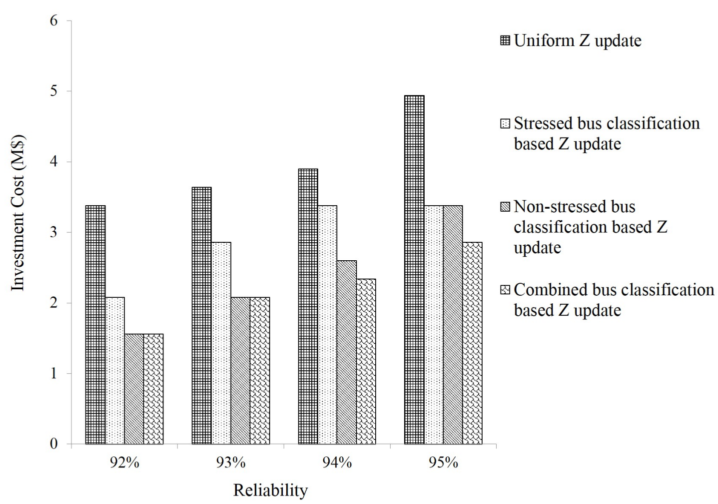

Figure 3: Variation in investment cost for IEEE 30-bus system.

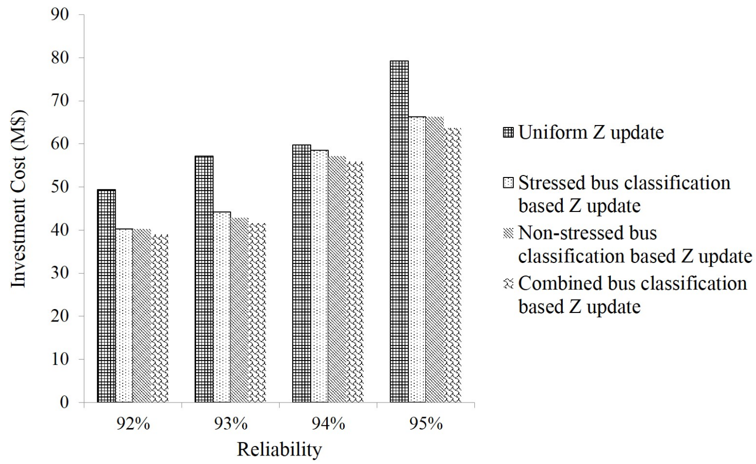

Figure 4: Variation in investment cost for IEEE 118-bus system.

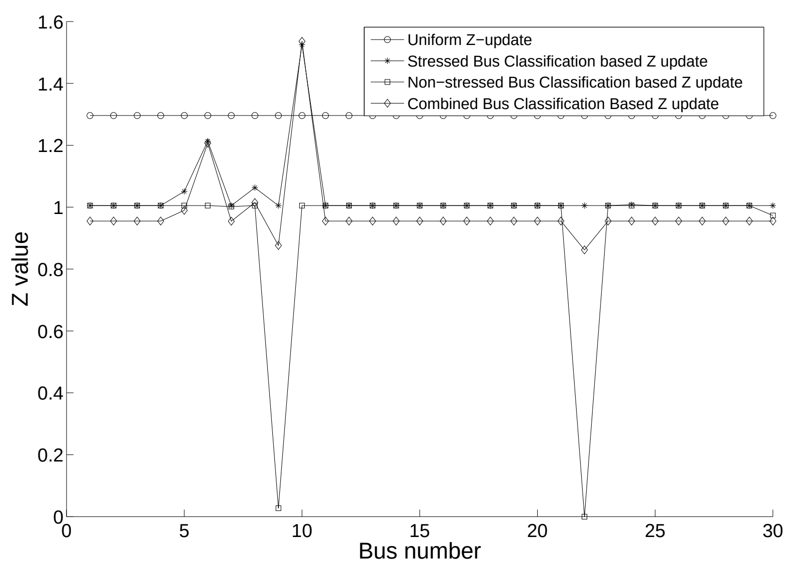

Figure 5: Z compared for IEEE 30-bus system and 92% target probability.

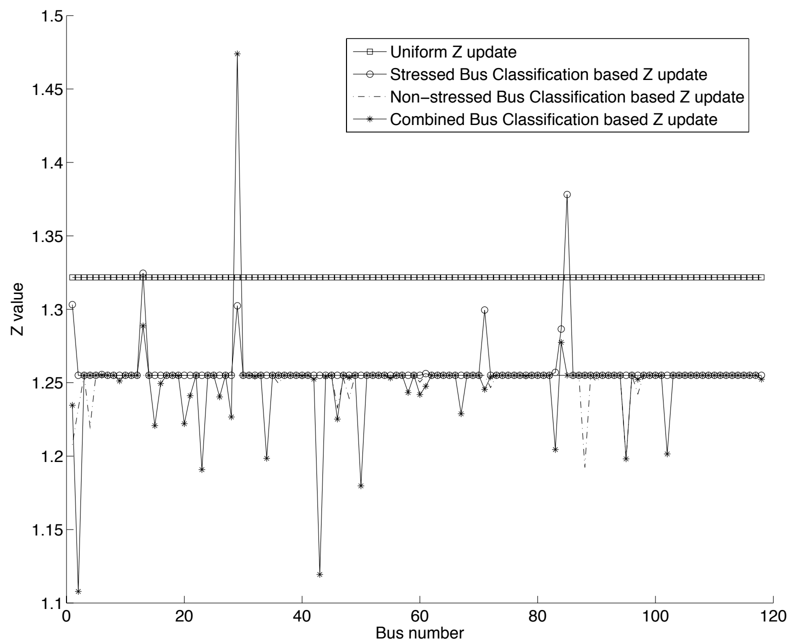

Figure 6: Z compared for IEEE 118-bus system and 92% target probability.

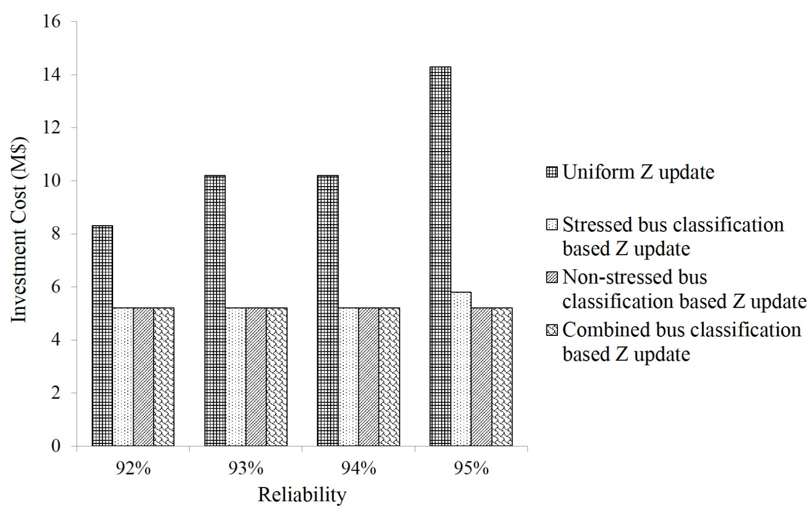

Figure 7: Comparison of investment cost for GEP considering different generation technologies for IEEE 30-bus test system.

Table 1: Variation in number of new units installed for IEEE 30-bus test system using different Z-update approaches.

α	Stressed-bus classification–based Z update		Nonstressed-bus classification– based Z update		Combined-bus classification–based Z update	
	No. of new units {5MW,3MW,1MW}	Investment cost (M$)	No. of new units {5MW,3MW,1MW}	Investment cost (M$)	No. of new units {5MW,3MW,1MW}	Investment cost (M$)
0.92	{1,1,1}	2.08	{1,0,1}	1.56	{1,0,1}	1.56
0.93	{2,0,1}	2.86	{1,1,0}	2.08	{1,1,0}	2.08
0.94	{2,1,0}	3.38	{2,0,0}	2.60	{1,1,1}	2.34
0.95	{2,1,0}	3.38	{2,1,0}	3.38	{2,0,1}	2.86

Table 2: Variation in number of new units installed for IEEE 118-bus test system using different Z-update approaches.

α	Stressed-bus classification–based Z update		Nonstressed-bus classification– based Z update		Combined-bus- classification–based Z update	
	No. of new 5 MW units	Investment cost (M$)	No. of new 5 MW units	Investment cost (M$)	No. of new 5 MW units	Investment cost (M$)
0.92	31	40.3	31	40.3	30	39.0
0.93	34	44.2	33	42.9	32	41.6
0.94	45	58.5	44	57.2	43	55.9
0.95	51	66.3	50	65.0	49	63.7

Table 3: Variation in number of new units installed using different Z-update algorithms in IEEE 30-bus system and for different generation technologies {coal,gas}.

α	Uniform Z update		Stressed-bus classification–based Z update		Nonstressed-bus classification–based Z update		Combined-bus classification–based Z update	
	New units of 5 MW	Investment cost (M$)	New units of 5 MW	Investment cost (M$)	New units of 5 MW	Investment cost (M$)	New units of 5 MW	Investment cost (M$)
0.92	{5,3}	8.3	{4,0}	5.2	{4,0}	5.2	{4,0}	5.2
0.93	{6,4}	10.2	{4,0}	5.2	{4,0}	5.2	{4,0}	5.2
0.94	{6,4}	10.2	{4,0}	5.2	{4,0}	5.2	{4,0}	5.2
0.95	{8,3}	14.3	{4,1}	5.8	{4,0}	5.2	{4,0}	5.2